LECTURE NO 20

Electrostatics

TOPIC COVERED

- Convection and conduction current
- Continuity equation
- Relaxation time

The **current** (in amperes) through a given area is the electric charge passing through the area per unit time.

That is,

$$I = \frac{dQ}{dt} \tag{5.1}$$

Thus in a current of one ampere, charge is being transferred at a rate of one columb per second.

We now introduce the concept of *current density* **J**. If current ΔI flows through a surface ΔS, the current density is

$$J_n = \frac{\Delta I}{\Delta S}$$

or

$$\Delta I = J_n \Delta S \tag{5.2}$$

assuming that the current density is perpendicular to the surface. If the current density is not normal to the surface,

$$\Delta I = \mathbf{J} \cdot \Delta \mathbf{S} \tag{5.3}$$

Thus, the total current flowing through a surface S is

$$I = \int_S \mathbf{J} \cdot d\mathbf{S} \tag{5.4}$$

Due to the principle of charge conservation, the time rate of decrease of charge within a given volume must be equal to the net outward current flow through the closed surface of the volume. Thus current I_{out} coming out of the closed surface is

$$I_{out} = \oint \mathbf{J} \cdot d\mathbf{S} = \frac{-dQ_{in}}{dt} \tag{5.40}$$

where Q_{in} is the total charge enclosed by the closed surface. Invoking divergence theorem

$$\oint_S \mathbf{J} \cdot d\mathbf{S} = \int_v \nabla \cdot \mathbf{J} \, dv \tag{5.41}$$

But

$$\frac{-dQ_{in}}{dt} = -\frac{d}{dt}\int_v \rho_v \, dv = -\int_v \frac{\partial \rho_v}{\partial t} dv \tag{5.42}$$

Substituting eqs. (5.41) and (5.42) into eq. (5.40) gives

$$\int_v \nabla \cdot \mathbf{J}\, dv = -\int_v \frac{\partial \rho_v}{\partial t}\, dv$$

or

$$\boxed{\nabla \cdot \mathbf{J} = -\frac{\partial \rho_v}{\partial t}} \tag{5.43}$$

$$\mathbf{J} = \sigma \mathbf{E} \tag{5.44}$$

and Gauss's law

$$\nabla \cdot \mathbf{E} = \frac{\rho_v}{\varepsilon} \tag{5.45}$$

Substituting eqs. (5.44) and (5.45) into eq. (5.43) yields

$$\nabla \cdot \sigma \mathbf{E} = \frac{\sigma \rho_v}{\varepsilon} = -\frac{\partial \rho_v}{\partial t}$$

or

$$\frac{\partial \rho_v}{\partial t} + \frac{\sigma}{\varepsilon} \rho_v = 0 \tag{5.46}$$

This is a homogeneous linear ordinary differential equation. By separating variables in eq. (5.46), we get

$$\frac{\partial \rho_v}{\rho_v} = -\frac{\sigma}{\varepsilon} \partial t \tag{5.47}$$

and integrating both sides gives

$$\ln \rho_v = -\frac{\sigma t}{\varepsilon} + \ln \rho_{vo}$$

where $\ln \rho_{vo}$ is a constant of integration. Thus

$$\boxed{\rho_v = \rho_{vo} e^{-t/T_r}} \tag{5.48}$$

where

$$\boxed{T_r = \frac{\varepsilon}{\sigma}} \tag{5.49}$$

Relaxation time is the time it takes a charge placed in the interior of a material to drop to e^{-1} = 36.8 percent of its initial value.